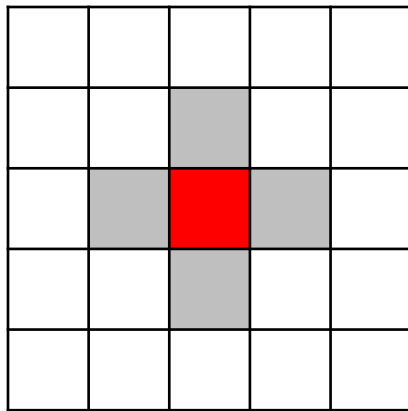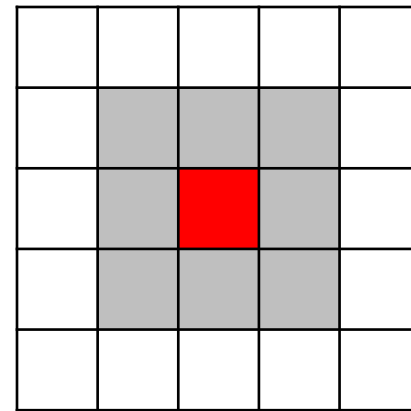# Day 25

Path Planning in Discrete Sampled Space

Connectivity in Discrete Sampled Space

▸ a path on a discrete grid is a sequence of moves between connected cells

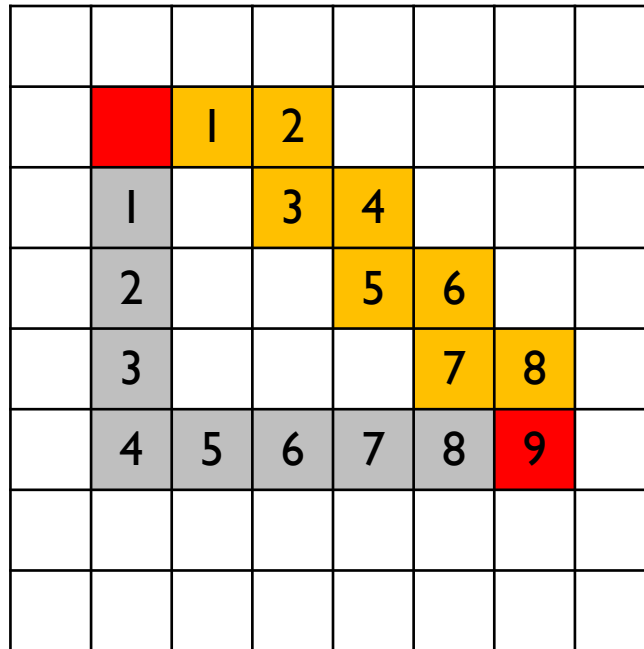▸ for a square tiling there are two possible definitions of connectivity

4-connectivity

8-connectivity

4-Connectivity

▸ on a 4-connected tiling the distance between two cells is called the taxicab distance, rectilinear distance, L_1 distance, L_1 norm, city block distance, or Manhattan distance

Wave-Front Planner

▸ the wave-front planner finds a path between a start and goal point in spaces represented as a grid where

  ▸ free space is labeled with a 0

  ▸ obstacles are labeled with a 1

  ▸ the goal is labeled with a 2

  ▸ the start is known

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

start

Wave-Front Planner

▸ starting at the goal cell

L := 2

while start cell is unlabelled

    for each cell C with label L

        for each cell Z connected to C with label 0

            label Z with L+1

    L := L + 1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

start

0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

start

0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	2
0	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	
0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

start

0	0	0	14	13	12	11	10	9	8	7	6	5	4	3	2
0	0	0	0	14	13	12	11	10	9	8	7	6	5	4	3
0	0	1	1	0	14	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

start

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
19	18	1	1	15	14	1	1	1	1	1	1	1	1	1	1
20	19	1	1	16	15	1	1	1	1	1	1	1	1	1	1
0	20	1	1	17	16	17	18	19	20	0	0	1	1	0	0
1	1	1	1	18	17	18	19	20	0	0	0	1	1	0	0
1	1	1	1	19	18	19	20	0	0	0	0	1	1	0	0
0	0	1	1	20	19	20	0	0	0	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

start

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
19	18	1	1	15	14	1	1	1	1	1	1	1	1	1	1
20	19	1	1	16	15	1	1	1	1	1	1	1	1	1	1
21	20	1	1	17	16	17	18	19	20	21	22	1	1	37	38
1	1	1	1	18	17	18	19	20	21	22	23	1	1	36	37
1	1	1	1	19	18	19	20	21	22	23	24	1	1	35	36
0	0	1	1	20	19	20	21	22	23	24	25	1	1	34	35
0	0	1	1	1	1	1	1	23	24	1	1	1	1	33	34
0	0	1	1	1	1	1	1	24	25	1	1	1	1	32	33
0	0	1	1	29	28	27	26	25	26	27	28	29	30	31	32
0	0	1	1	30	29	28	27	26	27	28	29	30	31	32	33
0	0	1	1	1	1	1	1	1	1	1	1	1	1	33	34
0	50	1	1	1	1	1	1	1	1	1	1	1	1	34	35
50	49	48	46	45	44	43	42	41	40	39	38	37	36	35	36
*	50	49	47	46	45	44	43	42	41	40	39	38	37	36	37

start

Wave-Front Planner

▸ to generate a path starting from the start point

L := start point label

while not at the goal

   move to any connected cell with label L-1

    L := L-1

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
19	18	1	1	15	14	1	1	1	1	1	1	1	1	1	1
20	19	1	1	16	15	1	1	1	1	1	1	1	1	1	1
21	20	1	1	17	16	17	18	19	20	21	22	1	1	37	38
1	1	1	1	18	17	18	19	20	21	22	23	1	1	36	37
1	1	1	1	19	18	19	20	21	22	23	24	1	1	35	36
0	0	1	1	20	19	20	21	22	23	24	25	1	1	34	35
0	0	1	1	1	1	1	1	23	24	1	1	1	1	33	34
0	0	1	1	1	1	1	1	24	25	1	1	1	1	32	33
0	0	1	1	29	28	27	26	25	26	27	28	29	30	31	32
0	0	1	1	30	29	28	27	26	27	28	29	30	31	32	33
0	51	1	1	1	1	1	1	1	1	1	1	1	1	33	34
51	50	1	1	1	1	1	1	1	1	1	1	1	1	34	35
50	49	48	46	45	44	43	42	41	40	39	38	37	36	35	36
51	50	49	47	46	45	44	43	42	41	40	39	38	37	36	37

start

3/13/2011

Wave-Front Planner

▶ advantage:

  ▶ will find a shortest path (in terms of connectivity) between start and goal if a path exists

  ▶ generalizes to higher dimensions

▶ disadvantages:

  ▶ path often runs adjacent to obstacles

  ▶ planner searches the entire space with radius R around the goal (where R is the distance between the start and goal)

  ▶ paths restricted by grid connectivity are longer than necessary

Wave-Front Planner

▸ paths restricted by grid connectivity are longer than necessary

　▸ Manhattan distance = 9

　▸ straight line distance = sqrt(16 + 25) = 6.403…
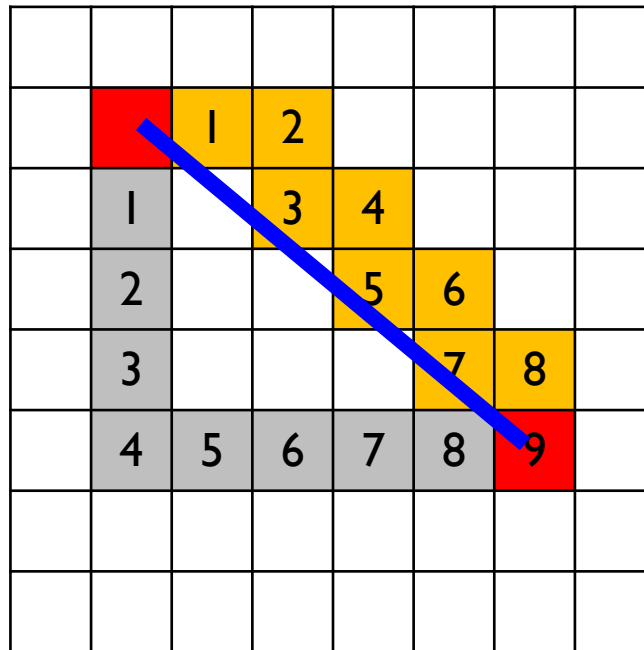
Planners Using Graph Algorithms

▸ A* is a common algorithm in game AI programming and robotics

  ▸ first described in 1968

  ▸ http://theory.stanford.edu/~amitp/GameProgramming/

▸ A* is the foundation for Theta*

  ▸ Daniel, Nash, Koenig. Theta*: Any-Angle Planning on Grids, Journal of Artificial Intelligence Research, 39, 2010.

  ▸ path planning on a grid where paths are allowed to pass through cells at any angle (not just using 4- or 8-connectivity)

Potential Functions

▸ in continuous space potential functions can be used for path planning

▸ a potential function is a differentiable real-valued function U

$$U : \mathrm{R}^m \to \mathrm{R}$$

　　▸ i.e., U assigns a scalar real value to every point in space

▸ potential functions you might know
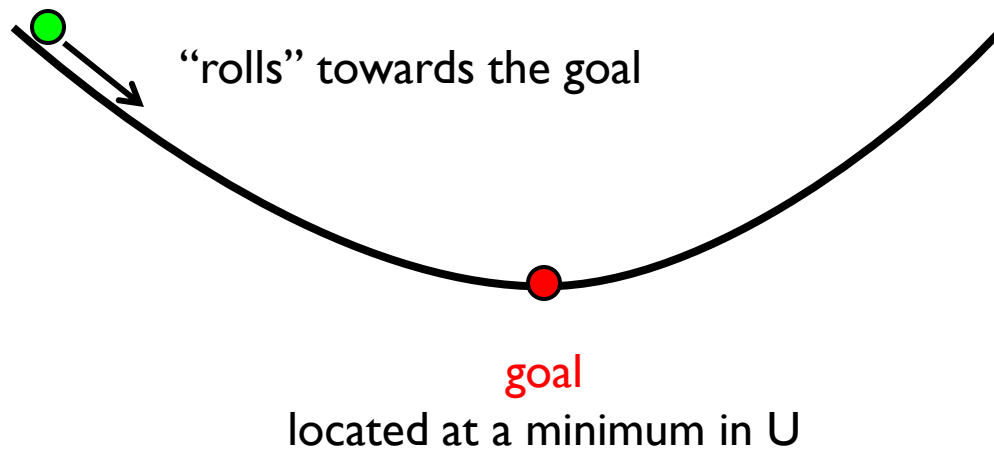
　　▸ gravitational potential

　　▸ electrostatic potential

Goal Potential

▸ the goal potential should be an attractive potential

  ▸ small near the goal

  ▸ large far from the goal

  ▸ monotonically increasing

    ▸ nice too if it is continuously differentiable

Goal Potential

▸ consider the quadratic potential

$$U_{\text{attract}} = \alpha \left\| q - q_{\text{goal}} \right\|^2$$

"rolls" towards the goal

goal
located at a minimum in U

Goal Potential

▸ "rolling towards the goal" can be accomplished using gradient descent

$$F = \nabla U_{\text{attract}}$$

$$= \begin{bmatrix} \partial U / \partial x \\ \partial U / \partial y \end{bmatrix}$$

$$= \alpha \left(q - q_{\text{goal}} \right)$$

▸ gradient descent

  ▸ starting at initial configuration, take a small step in the direction opposite to the gradient F until $|F| = 0$

Goal Potential

▸ notice that the wave-front planner basically works this way

  ▸ it defines a potential where there is only one minimum

    ▸ the minimum is located at the goal

  ▸ it then uses gradient descent to move towards the goal

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
19	18	1	1	15	14	1	1	1	1	1	1	1	1	1	1
20	19	1	1	16	15	1	1	1	1	1	1	1	1	1	1
21	20	1	1	17	16	17	18	19	20	21	22	1	1	37	38
1	1	1	1	18	17	18	19	20	21	22	23	1	1	36	37
1	1	1	1	19	18	19	20	21	22	23	24	1	1	35	36
0	0	1	1	20	19	20	21	22	23	24	25	1	1	34	35
0	0	1	1	1	1	1	1	23	24	1	1	1	1	33	34
0	0	1	1	1	1	1	1	24	25	1	1	1	1	32	33
0	0	1	1	29	28	27	26	25	26	27	28	29	30	31	32
0	0	1	1	30	29	28	27	26	27	28	29	30	31	32	33
0	51	1	1	1	1	1	1	1	1	1	1	1	1	33	34
51	50	1	1	1	1	1	1	1	1	1	1	1	1	34	35
50	49	48	46	45	44	43	42	41	40	39	38	37	36	35	36
51	50	49	47	46	45	44	43	42	41	40	39	38	37	36	37

start